

Complete time-resolved reconstruction of cavity field quantum states

Équipe: Électrodynamique quantique en cavité:

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Aim of the experiment

- Complete reconstruction of quantum states of trapped light fields.
- Observe time evolution of the cavity field and study its decoherence by performing time-resolved state reconstruction.

Methods

- Quantum non-demolition (QND) measurement of the photon number distribution after displacing a cavity field providing information on a whole density operator ρ .
- Reconstruction of ρ from measured raw data using maximum entropy principle.

Results

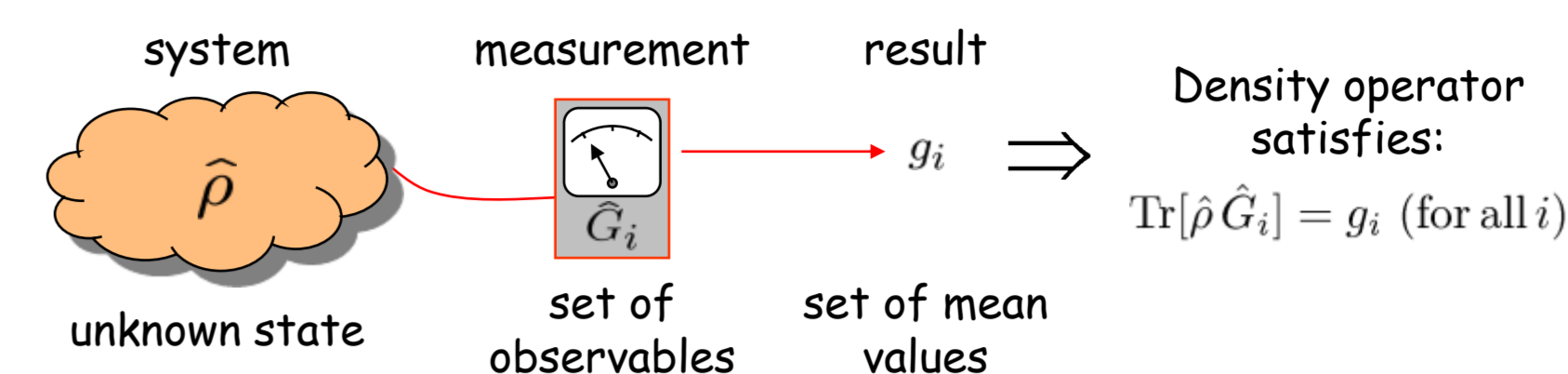
- Reconstruction of various field states illustrated by classical states, photon-number states and quantum superpositions of classical states ("Schrödinger's cat" states).
- Real-time observation of decoherence of the cat states revealing their fast evolution from initial quantum superposition to final classical mixture.

Reference

S. Deléglise *et al.*, Nature **455**, 510 (2008)

Quantum state measurement

Being a statistical concept, the **state** cannot be inferred from a single system's realization, but can be reconstructed from an **ensemble** of copies by measuring non-commuting observables on different realizations.



General principle:

- QND of photon number \Rightarrow information only on **diagonal** elements of a density operator ρ (in a photon-number basis)
- QND after **displacing** ρ in a phase-space (i.e. controlled mixing of all matrix elements) \Rightarrow **off-diagonal** elements of ρ

Wigner function

a quasi-probability distribution in a phase-space (x,p) :

$$W(\alpha) = \frac{2}{\pi} \text{Tr} [\hat{\rho} \hat{D}(\alpha) e^{i\pi \hat{a}^\dagger \hat{a}} \hat{D}(-\alpha)], \quad (\alpha = x + ip)$$

Important properties of Wigner function

- completely characterizes a quantum state
- is easily adapted for a mode of an electro-magnetic field
- is negative for non-Gaussian non-classical states

Measurement operator:

"Generalized parity" operator in the displaced phase space

$$\hat{G}_i = \hat{D}(\alpha_i) \hat{T} \hat{D}(-\alpha_i)$$

Measurement procedure:

- > **Displace** the cavity field by injecting in coherent field α_i :

$$\hat{\rho}^{(\alpha_i)} = \hat{D}(-\alpha_i) \hat{\rho} \hat{D}(\alpha_i)$$

$$\hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a})$$

- > Repeated QND measurement on the displaced field:

$$g_i = \text{Tr}[\hat{\rho}^{(\alpha_i)} \hat{T}]$$

$$\hat{T} = \cos(\eta + \Phi(\hat{N}))$$

Schrödinger's cat states

Preparation of phase cat states

1. Inject a small coherent field into the cavity:

$$|\beta\rangle = \frac{1}{2}(|\beta\rangle + |-\beta\rangle) + \frac{1}{2}(|\beta\rangle - |-\beta\rangle)$$

even n odd n

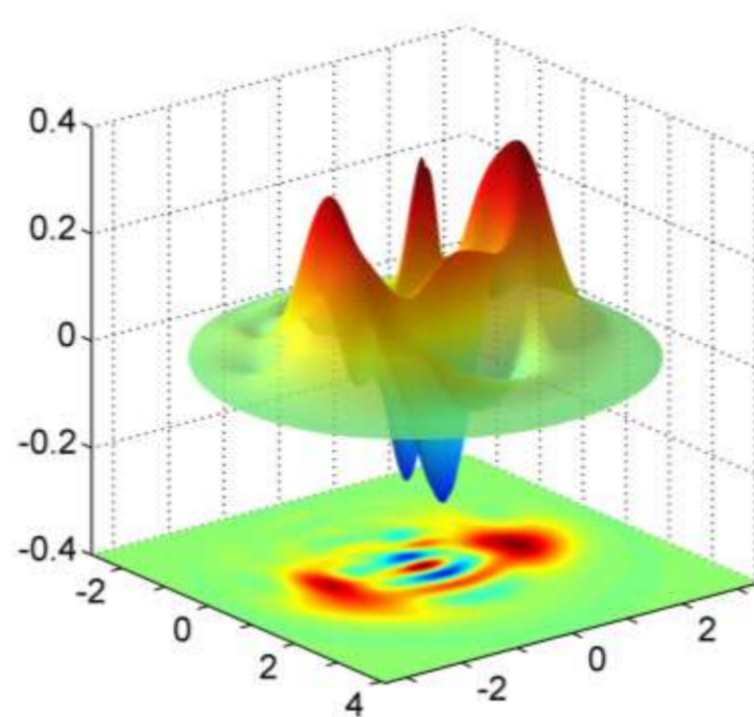
Eigenstates of parity

2. Measure the field parity with one atom: $|e\rangle \rightarrow$ "odd" field
 $|g\rangle \rightarrow$ "even" field

\Rightarrow Parity measurement projects a coherent field onto a cat state:

$$\Psi_{\text{even}} = \frac{1}{N}(|\beta\rangle + |-\beta\rangle) \quad \text{or} \quad \Psi_{\text{odd}} = \frac{1}{N'}(|\beta\rangle - |-\beta\rangle)$$

Wigner function – state tomography



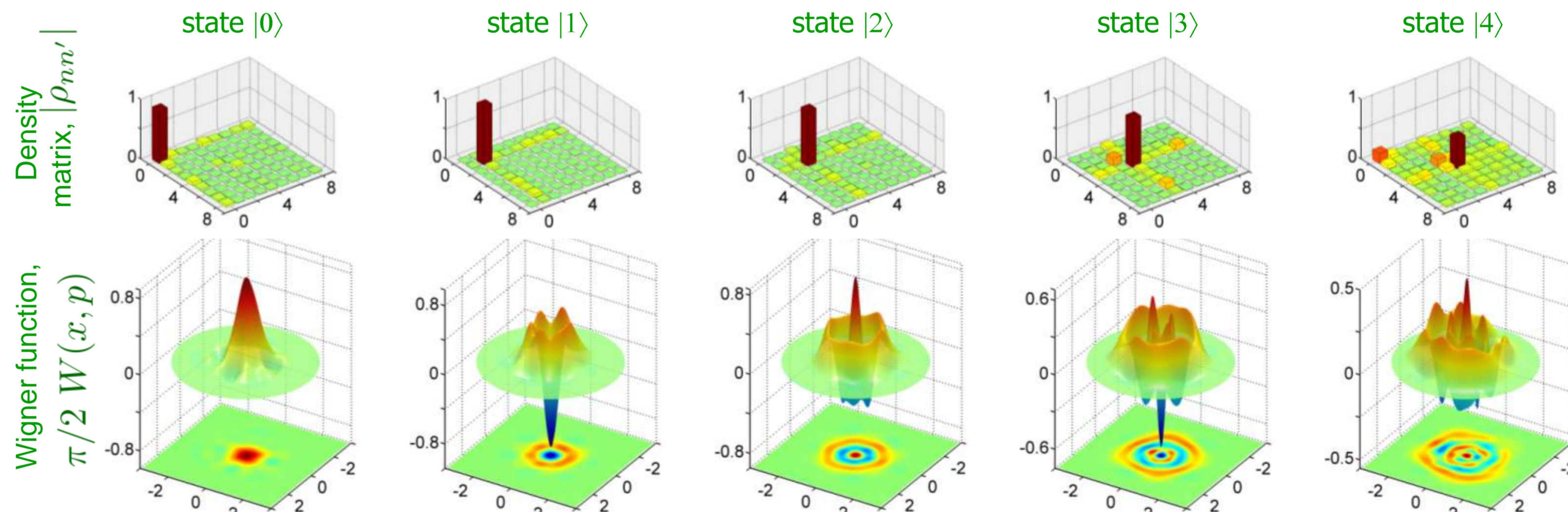
Fock states

Reconstruction parameters:

- mean photon number per photon $\phi = 0.5 \pi$
- two Ramsey phases $\eta = 0$ and $\pi/2$, sensitive to even and odd n

Wigner function properties:

- number of rings = number of photons
- value @ origin = state parity



Measurement of cat states

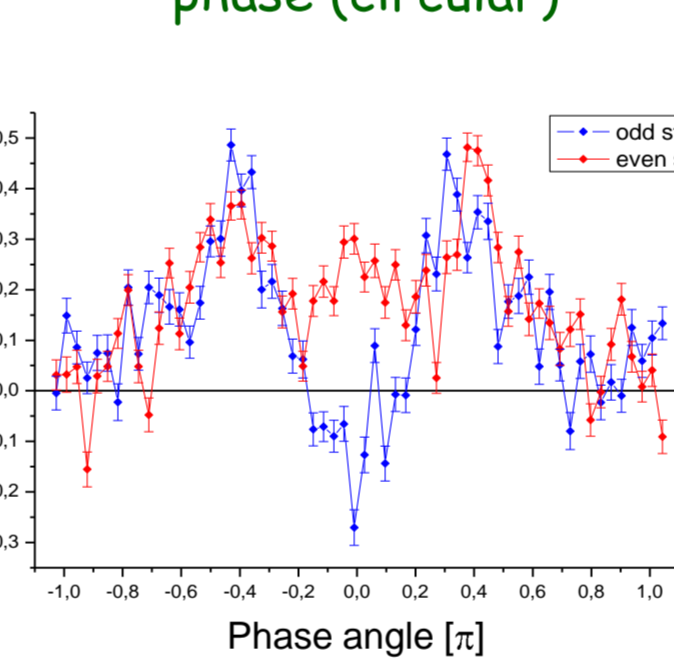
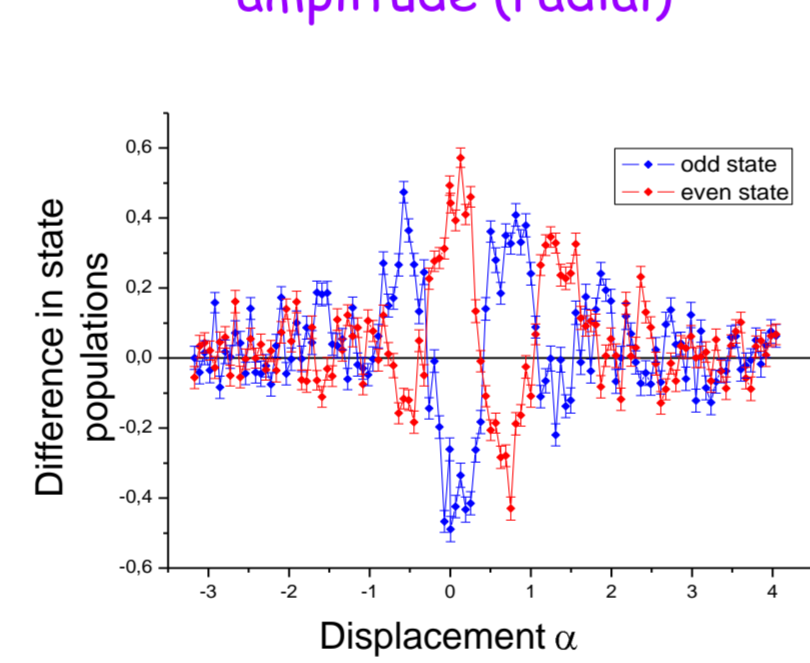
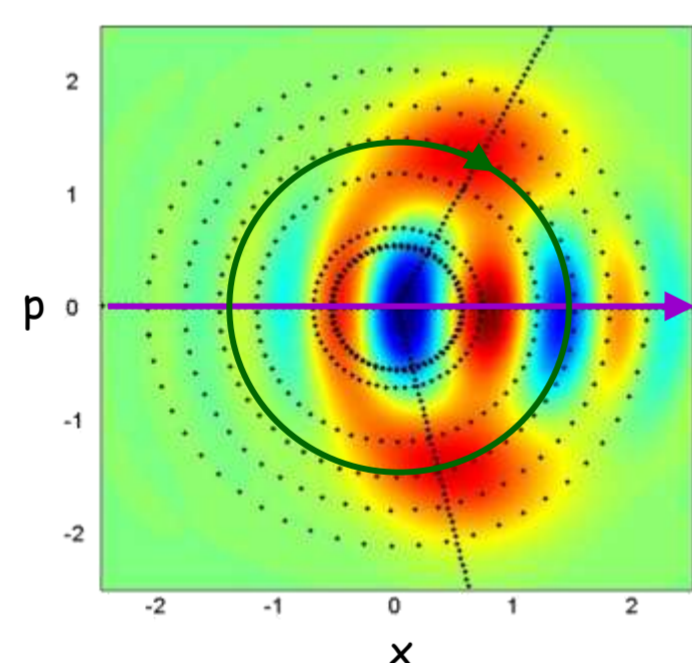
Two ways to scan the field displacement:

amplitude (radial)

phase (circular)

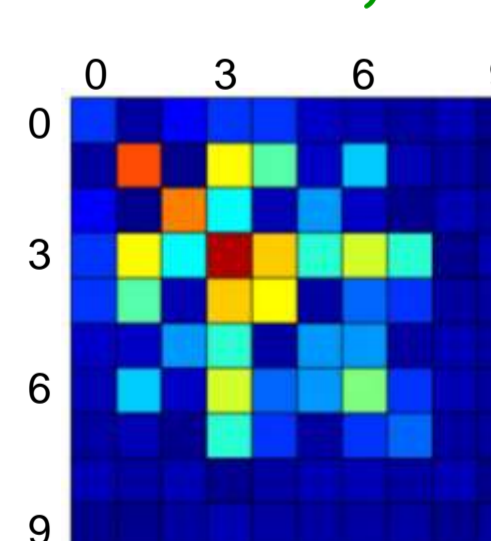
Expected distribution in phase-space:

- > Amplitudes of coherent fields $\beta = 3.5$
- > Their angular separation $2\phi = 0.8 \pi$
- > Cat size (separation of coherent components) $D^2 = 11$ photons



Coherence measure

Reconstructed ρ (absolute value, odd cat)



Translated ρ

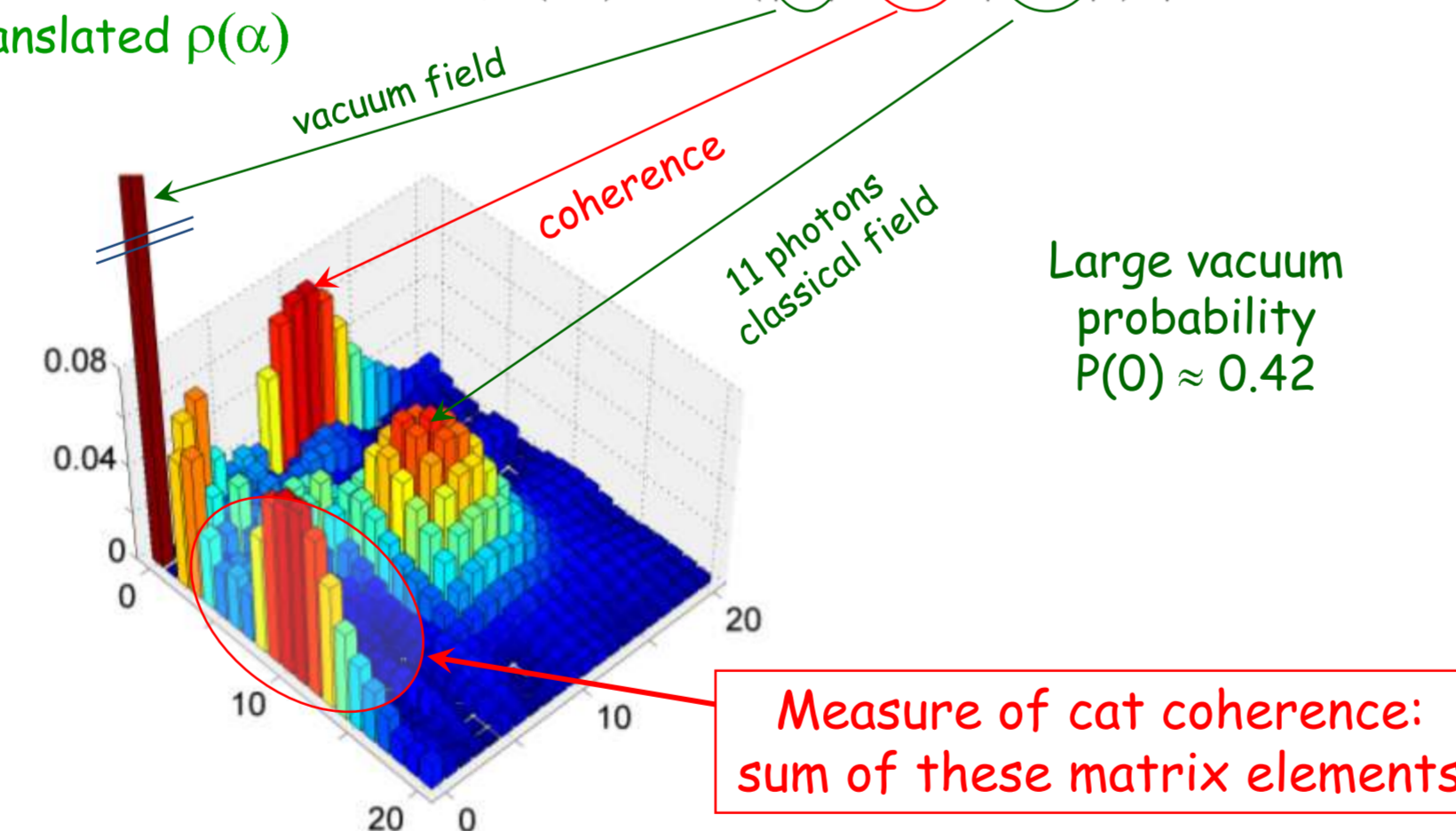
mathematical transformation

$$\hat{\rho}(\alpha) = \hat{D}(\alpha) \hat{\rho} \hat{D}(-\alpha)$$

α is adjusted to shift one classical component of the cat to vacuum

Absolute value of the translated $\rho(\alpha)$

$$\hat{\rho}(\alpha) = (|0\rangle + |2\alpha\rangle) / \sqrt{2}$$



Decoherence of cat states

Interaction with **environment** leads to mixing of the cat states: The information on quantum phase gets completely lost, resulting in **decoherence**.

Although a random **loss** of one photon does not change a coherent field, it changes the parity of a cat state, inducing its **decay**.

Cavity damping

$$T_{\text{dec}} = \frac{2T_{\text{cav}}}{D_{\text{cat}}^2} \leftarrow \text{cavity decay time}$$

\leftarrow cat size

$$T_{\text{cav}} = 130 \text{ ms}$$

$$D_{\text{cat}}^2 \approx 11.8 \text{ ph} \Rightarrow T_{\text{dec}} \approx 22 \text{ ms}$$

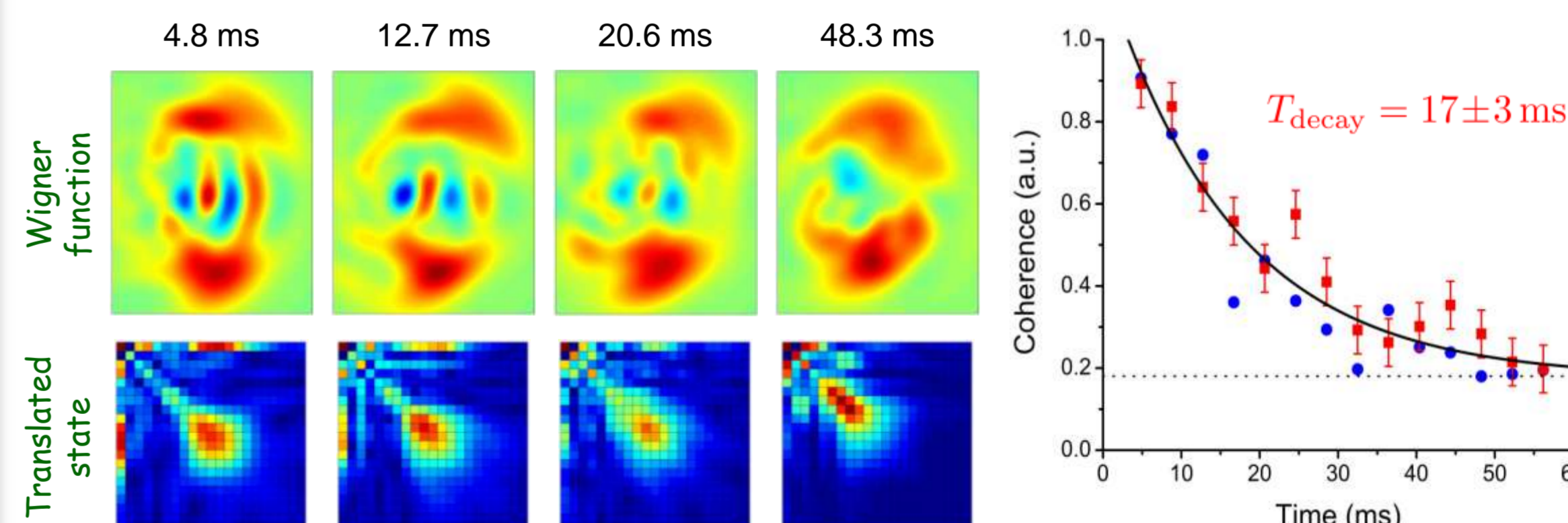
Coupling to thermal bath

$$T_{\text{dec}, \bar{n}_{\text{th}}} = \left(1 + \frac{4\bar{n}_{\text{th}}}{D_{\text{cat}}^2} + 2\bar{n}_{\text{th}}\right)^{-1} T_{\text{dec}}$$

mean number of thermal photons

$$\bar{n}_{\text{th}} = 0.05 \text{ ph} \quad (@ 0.8 \text{ K}) \Rightarrow T_{\text{dec}, \bar{n}_{\text{th}}} \approx 19.5 \text{ ms}$$

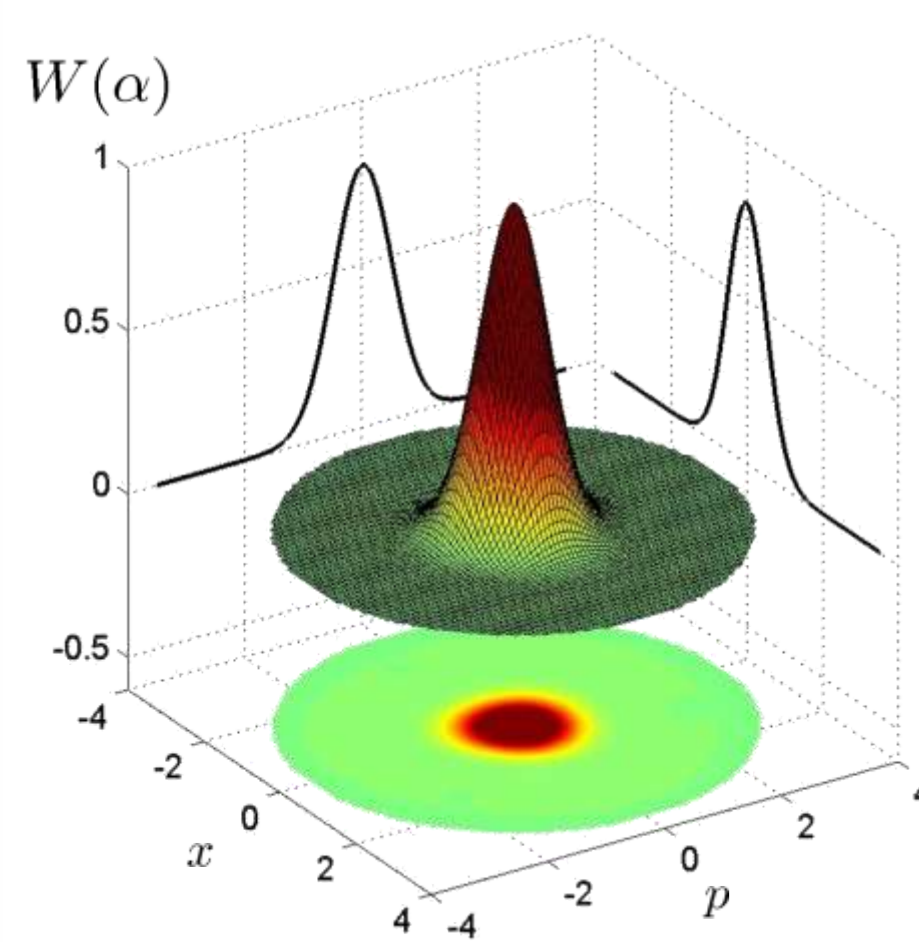
$$T_{\text{meas}} \approx 4 \text{ ms} < T_{\text{dec}, \bar{n}_{\text{th}}}$$



Step-by-step generation of phase cat states

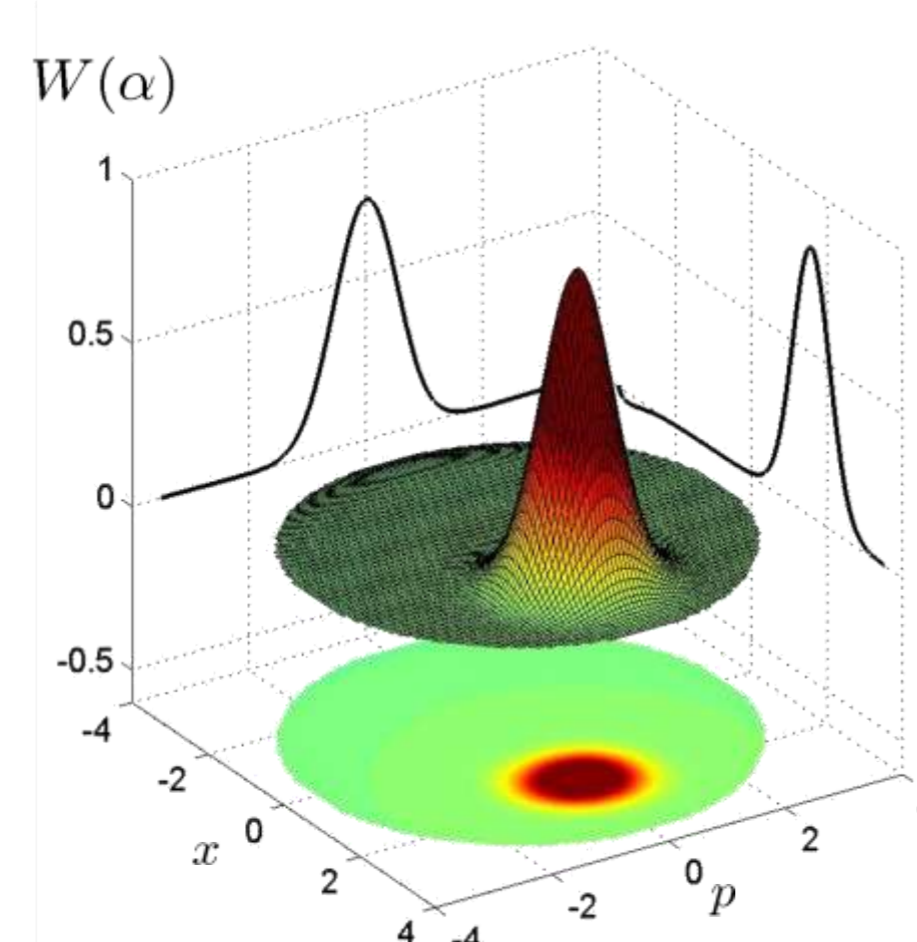
after sweeping the cavity with resonant atoms

vacuum state



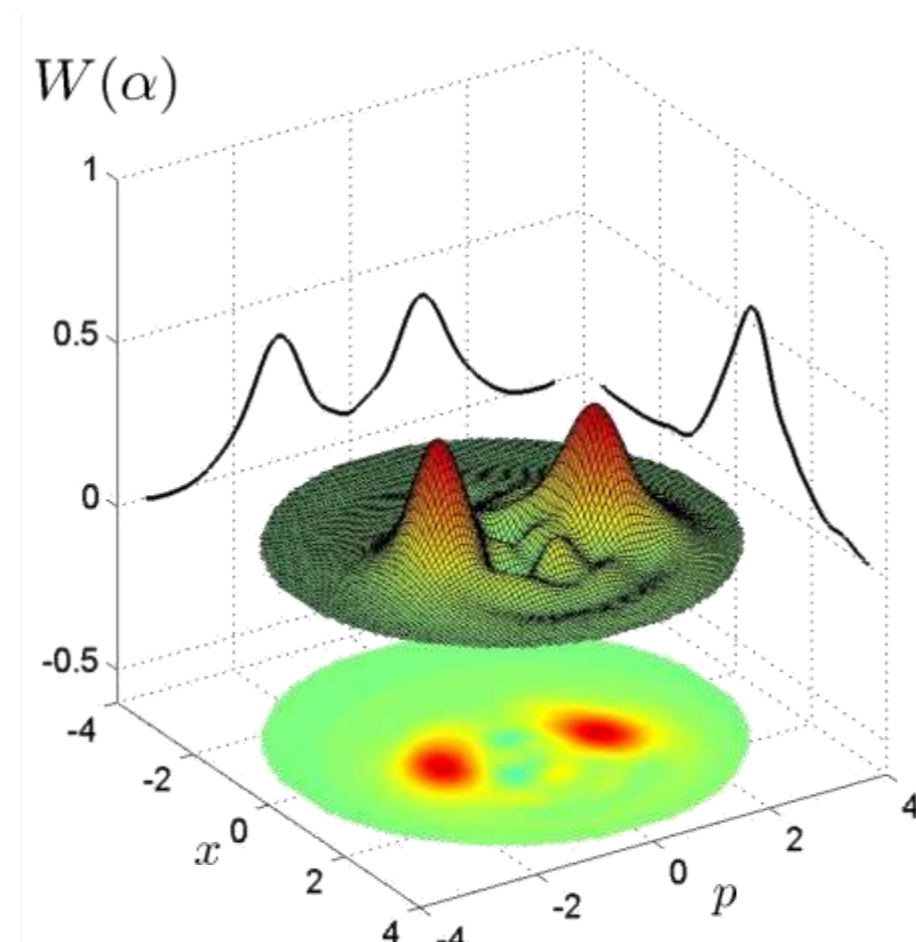
after MW injection of amplitude β

coherent state



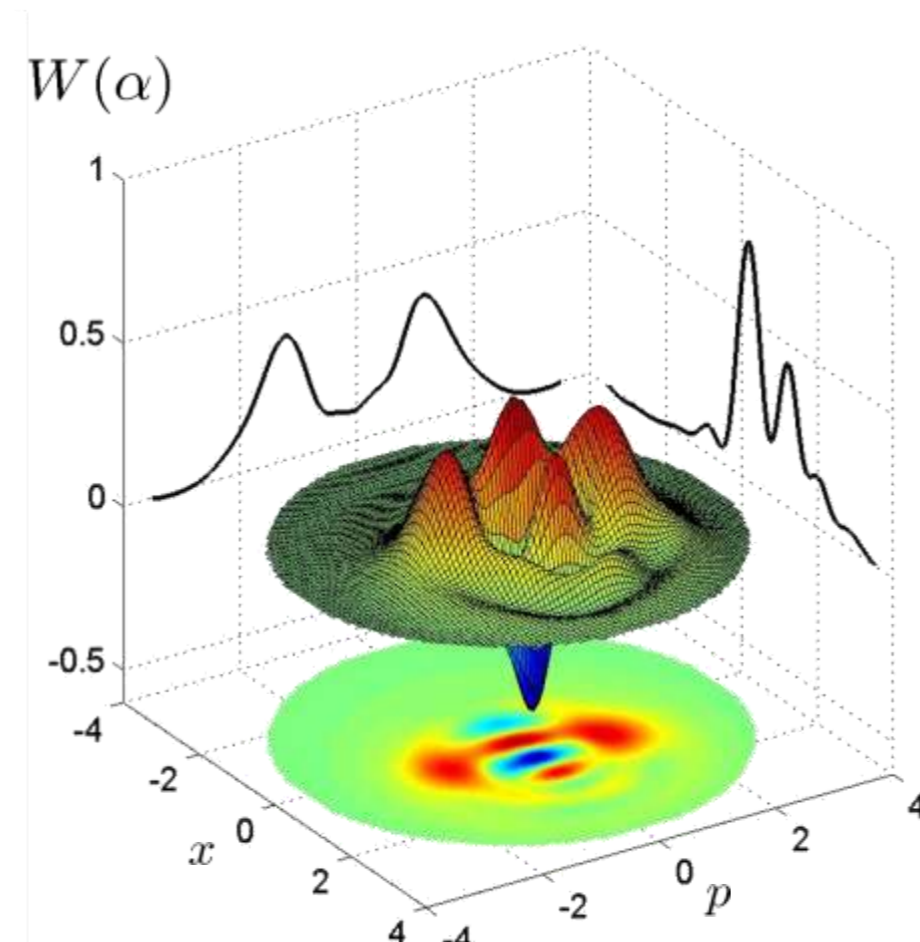
after interacting with an atom in the superposition state

statistical mixture

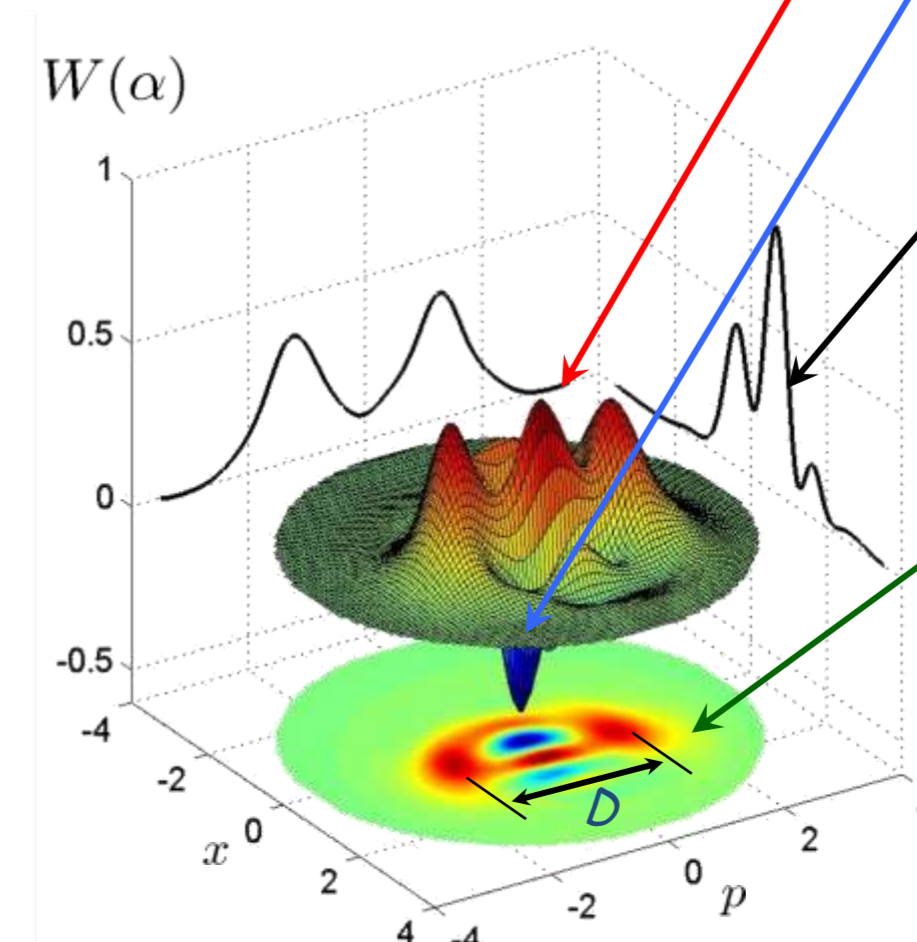


after detecting the atom's state, the field is projected onto one of two cat states

even cat state



odd cat state



Interference fringes \Rightarrow quantum superposition of classical fields

Negative values \Rightarrow quantum signature of the prepared state

Real and positive marginal distributions of field quadratures

As expected, both cat states have:

- about 2.1 photons in each coherent component (amplitude of the initial coherent field)
- an angular separation of 0.8π (chosen phase shift per photon)
- cat size $D^2 \approx 7.5$ photons
- completely separated classical parts ($D \gg 1$)